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HYDRODYNAMIC CHARACTERISTICS OF A VORTEX SOURCE PERFORMING TRANSLATIONAL MOTION IN A MULTILAYER HEAVY FLUID

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The problem of translational motion of a vortex source in a three-layer fluid bounded by a bottom from below is considered. The fluid in each layer is perfect, incompressible, heavy, and homogeneous. Based on the previously developed method, formulas for disturbed complex velocities of the fluid in each layer and the wave drag and lift force of the vortex source are obtained. The vortex motion is considered near the interface of two semi-infinite fluid media and in a two-layer fluid with different conditions at the boundary. In all cases, the hydrodynamic characteristics of the vortex source are given as functions of the Froude number. In a number of problems, these characteristics have discontinuities at the transition through the critical Froude numbers. The character of these discontinuities is studied analytically.

The most complete review of investigations of generation of surface and internal waves by a body moving in a fluid can be found in [1–3]. The problem of motion of a vortex source is the basic one in this field, since it allows an analytical solution. The motion of a vortex source was studied under a free surface [4, 5], near the interface of two fluids [6], under a free surface of a fluid of finite depth [7, 8], and in a two-layer fluid under a free surface [9, 10].

A large number of papers dealing with the solution of a more general problem of motion of a contour in a multilayer fluid have recently been published [11–19]. It was found that the hydrodynamic characteristics in some cases have discontinuities at the transition through the critical Froude numbers at which the character of wave generation become qualitatively different.

A method for solving linear problems of motion of a vortex source in a fluid with an arbitrary finite number of layers was developed in [20]. Dependences of the hydrodynamic characteristics on the parameters of the problem were studied in the problem of motion of a vortex source in two-layer and three-layer fluids in [21, 22]. Based on the method proposed in [20], it is of interest to consider a more general problem of motion of a vortex source in a three-layer fluid bounded by a bottom from below and find analytically the character of discontinuities in the vicinity of the critical Froude numbers. These studies are described in the present paper.

1. We consider a linear problem of translational motion of a vortex source of intensity $C = \Gamma + iQ$ in a three-layer fluid bounded by a bottom from below. The fluid consists of the layers D_1, D_2, and D_3 (D_1 is the lower layer). The Ox axis of the coordinate system is aligned with the undisturbed interface between the layers D_2 and D_3. The vortex source is located at the point $z_0 = x_0 - iy_0$ of the layer D_r ($r = 1, 2$).

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We introduce the following notation: g is the acceleration of gravity, V_∞ is the velocity of the fluid at infinity on the left, ρ_k is the fluid density in the layer D_k ($k = 1, 2, 3$), and H_1 and H_2 are the thicknesses of the layers $D_1 \cup D_2$ and D_2, respectively.

The disturbed motion of the fluid in the layer D_k is described by the complex velocity $\bar{V}_k(z)$ ($k = 1$, 2, 3). The functions $\bar{V}_k(z)$ are analytical in the layer D_k and satisfy the following boundary conditions [20]:

1) continuity of pressure at the transition through the interface between the media D_k and D_{k+1} ($k = 1, 2$):

$$\operatorname{Re}\Big\{ m_{k\,k+1}^{k} \frac{d\bar{V}_k(z)}{dz} - m_{k\,k+1}^{k+1} \frac{d\bar{V}_{k+1}(z)}{dz} + i\nu_k \bar{V}_k(z) \Big\} = 0, \quad z = x + iH_2(k-2), \tag{1.1}$$

$$m_{k\,k+1}^{k} = \frac{\rho_k}{\rho_k + \rho_{k+1}}, \quad m_{k\,k+1}^{k+1} = \frac{\rho_{k+1}}{\rho_k + \rho_{k+1}}, \quad m_{k\,k+1} = \frac{\rho_k - \rho_{k+1}}{\rho_k + \rho_{k+1}}, \quad \nu = \frac{g}{V_\infty^2}, \quad \nu_k = \nu m_{k\,k+1};$$

2) continuity of the normal component of velocity at the transition through the interfaces between the media D_k and D_{k+1} ($k = 1, 2$):

$$\operatorname{Im}\{\bar{V}_k(z) - \bar{V}_{k+1}(z)\} = 0, \qquad z = x + iH_2(k-2); \tag{1.2}$$

3) zero normal component of velocity at the bottom of the fluid:

$$\operatorname{Im}\{\bar{V}_1(z)\} = 0, \qquad z = x - iH_1; \tag{1.3}$$

4) decay of velocity disturbances at infinity on the left:

$$\lim_{x \to -\infty} \bar{V}_k(z) = 0 \qquad (k = 1, 2, 3). \tag{1.4}$$

Using the generic method for solving linear problems of motion of a vortex source in a multilayer fluid [20], we obtain the solution of the boundary-value problem (1.1)–(1.4):

$$\bar{V}_1(z) = \frac{C}{2\pi i} \frac{2-r}{z - z_0} + \frac{1}{\pi} \int_0^\infty G_1^r(\lambda) e^{-i\lambda(z - \bar{z}_0)}\, d\lambda - i \sum_{j=1}^{P} \operatorname*{Res}_{\lambda=\lambda_j} (G_1^r(\lambda) e^{-i\lambda(z-\bar{z}_0)})$$

$$+ \frac{1}{\pi} \int_0^\infty G_2^r(\lambda) e^{i\lambda(z - z_0)}\, d\lambda + i \sum_{j=1}^{P} \operatorname*{Res}_{\lambda=\lambda_j} (G_2^r(\lambda) e^{i\lambda(z-z_0)}); \tag{1.5}$$

$$\bar{V}_2(z) = \frac{C}{2\pi i} \frac{r-1}{z - z_0} + \frac{1}{\pi} \int_0^\infty G_3^r(\lambda) e^{-i\lambda(z - \bar{z}_0)}\, d\lambda - i \sum_{j=1}^{P} \operatorname*{Res}_{\lambda=\lambda_j} (G_3^r(\lambda) e^{-i\lambda(z-\bar{z}_0)})$$

$$+ \frac{1}{\pi} \int_0^\infty G_4^r(\lambda) e^{i\lambda(z - z_0)}\, d\lambda + i \sum_{j=1}^{P} \operatorname*{Res}_{\lambda=\lambda_j} (G_4^r(\lambda) e^{i\lambda(z-z_0)}); \tag{1.6}$$

$$\bar{V}_3(z) = \frac{1}{\pi} \int_0^\infty G_5^r(\lambda) e^{i\lambda(z - z_0)}\, d\lambda + i \sum_{j=1}^{P} \operatorname*{Res}_{\lambda=\lambda_j} (G_5^r(\lambda) e^{i\lambda(z-z_0)}). \tag{1.7}$$

Here

$$G_1^1(\lambda) = \mathrm{e}^{2\lambda H_2}(\bar{C} - C\mathrm{e}^{2\lambda(y_0-H_1)})((\lambda m_{23} + \nu_2)(\lambda + \nu_1) + (\lambda - \nu_2)(\lambda m_{12} + \nu_1)\mathrm{e}^{2\lambda H_2})/2T(\lambda),$$

$$G_2^1(\lambda) = \mathrm{e}^{-2\lambda H_1}(\bar{C}((\lambda m_{23} + \nu_2)(\lambda m_{12} - \nu_1) + (\lambda - \nu_2)(\lambda - \nu_1)\mathrm{e}^{2\lambda H_2})\mathrm{e}^{2\lambda y_0}$$
$$+ C((\lambda m_{23} + \nu_2)(\lambda + \nu_1) + (\lambda - \nu_2)(\lambda m_{12} + \nu_1)\mathrm{e}^{2\lambda H_2})\mathrm{e}^{2\lambda H_2})/2T(\lambda),$$

$$G_3^1(\lambda) = \lambda m_{12}^1(\lambda m_{23} + \nu_2)\mathrm{e}^{2\lambda H_2}(\bar{C} - C\mathrm{e}^{2\lambda(y_0-H_1)})/T(\lambda),$$

$$G_4^1(\lambda) = \lambda m_{12}^1(\lambda - \nu_2)\mathrm{e}^{2\lambda H_2}(\bar{C}\mathrm{e}^{2\lambda(y_0-H_1)} - C)/T(\lambda),$$

$$G_5^1(\lambda) = 2m_{12}^1 m_{23}^2 \lambda^2 \mathrm{e}^{2\lambda H_2}(\bar{C}\mathrm{e}^{2\lambda(y_0-H_1)} - C)/T(\lambda),$$

$$G_1^2(\lambda) = m_{12}^2 \lambda\, \mathrm{e}^{2\lambda H_2}(\bar{C}(\lambda m_{23} + \nu_2) + C(\lambda - \nu_2)\mathrm{e}^{2\lambda y_0})/T(\lambda),$$

$$G_2^2(\lambda) = m_{12}^2 \lambda\, \mathrm{e}^{2\lambda(H_2-H_1)}(\bar{C}(\lambda - \nu_2)\mathrm{e}^{2\lambda y_0} + C(\lambda m_{23} + \nu_2))/T(\lambda),$$

$$G_3^2(\lambda) = (\lambda m_{23} + \nu_2)(\bar{C}\mathrm{e}^{2\lambda H_2}(\lambda - \nu_1 + (\lambda m_{12} + \nu_1)\mathrm{e}^{2\lambda(H_2-H_1)})$$
$$- C\mathrm{e}^{2\lambda y_0}(\lambda m_{12} - \nu_1 + (\lambda + \nu_1)\mathrm{e}^{2\lambda(H_2-H_1)}))/2T(\lambda),$$

$$G_4^2(\lambda) = (\bar{C}(\lambda - \nu_2)\mathrm{e}^{2\lambda y_0} + C(\lambda m_{23} + \nu_2))(\lambda m_{12} - \nu_1 + (\lambda + \nu_1)\mathrm{e}^{2\lambda(H_2-H_1)})/2T(\lambda),$$

$$G_5^2(\lambda) = m_{23}^2 \lambda(\bar{C}(\lambda m_{12} - \nu_1 + (\lambda + \nu_1)\mathrm{e}^{2\lambda(H_2-H_1)})\mathrm{e}^{2\lambda y_0}$$
$$- C(\lambda - \nu_1 + (\lambda m_{12} + \nu_1)\mathrm{e}^{2\lambda(H_2-H_1)})\mathrm{e}^{2\lambda H_2})/T(\lambda),$$

$$T(\lambda) = (\lambda m_{12} - \nu_1)(\lambda m_{23} + \nu_2) + (\lambda - \nu_1)(\lambda - \nu_2)\mathrm{e}^{2\lambda H_2}$$
$$+ ((\lambda + \nu_1)(\lambda m_{23} + \nu_2) + (\lambda m_{12} + \nu_1)(\lambda - \nu_2)\mathrm{e}^{2\lambda H_2})\mathrm{e}^{2\lambda(H_2-H_1)}.$$

Here λ_j ($j = 1, \ldots, P$) are the positive roots of the equation $T(\lambda) = 0$. An analysis of this equation shows that it has two roots for $\mathrm{Fr} < \mathrm{Fr}_{1*}$, one root for $\mathrm{Fr}_{1*} < \mathrm{Fr} < \mathrm{Fr}_{2*}$, and no roots for $\mathrm{Fr} > \mathrm{Fr}_{2*}$ [$\mathrm{Fr} = V_\infty^2/(gH_2)$ is the Froude number]. The critical Froude numbers Fr_{1*} and Fr_{2*} are determined as $\mathrm{Fr}_{1,2*} = 1/(\nu_{1,2*}H_2)$, where ν_{1*} and ν_{2*} are the roots of the equation

$$m_{12}m_{23}H_2(H_1 - H_2)\nu^2 + (m_{12}m_{23}^3 H_2 - (m_{12}m_{23}^2 + m_{12}^2 m_{23})H_1)\nu + m_{12}^1 m_{23}^2 = 0. \tag{1.8}$$

Using the generic method of [20], we obtain the following expressions for the hydrodynamic characteristics of the vortex source located in the layer D_r ($r = 1, 2$):

$$R_x^r = -\rho_r Q V_\infty + \Delta R_x^r, \qquad R_y^r = -\rho_r \Gamma V_\infty + \Delta R_y^r;$$

$$\Delta R_x^1 = \sum_{j=1}^{P} \frac{\rho_1}{T'(\lambda_j)} \Big[\frac{1}{2}\, \mathrm{e}^{2\lambda_j(H_2-y_0)}(\Gamma^2 + Q^2)((\lambda_j + \nu_1)(\lambda_j m_{23} + \nu_2) + (\lambda_j m_{12} + \nu_1)(\lambda_j - \nu_2)\mathrm{e}^{2\lambda_j H_2})$$
$$- \mathrm{e}^{2\lambda_j(H_2-H_1)}(\Gamma^2 - Q^2)((\lambda_j + \nu_1)(\lambda_j m_{23} + \nu_2) + (\lambda_j m_{12} + \nu_1)(\lambda_j - \nu_2)\mathrm{e}^{2\lambda_j H_2})$$
$$- \frac{1}{2}\, \mathrm{e}^{2\lambda_j(y_0-H_1)}(\Gamma^2 + Q^2)((\lambda_j m_{12} - \nu_1)(\lambda_j m_{23} + \nu_2) + (\lambda_j - \nu_1)(\lambda_j - \nu_2)\mathrm{e}^{2\lambda_j H_2})\Big]; \tag{1.9}$$

$$\Delta R_y^1 = -\frac{\rho_1}{2\pi}(\Gamma^2+Q^2)\int\limits_0^\infty [((\lambda+\nu_1)(\lambda m_{23}+\nu_2)+(\lambda m_{12}+\nu_1)(\lambda-\nu_2)e^{2\lambda H_2})e^{2\lambda(H_2-y_0)}$$

$$+((\lambda m_{12}-\nu_1)(\lambda m_{23}+\nu_2)+(\lambda-\nu_1)(\lambda-\nu_2)e^{2\lambda H_2})e^{2\lambda(y_0-H_1)}]\frac{d\lambda}{T(\lambda)}; \quad (1.10)$$

$$\Delta R_x^2 = \sum_{j=1}^{P}\frac{\rho_2}{T'(\lambda_j)}\Big[-(\lambda_j m_{23}+\nu_2)(\Gamma^2-Q^2)(\lambda_j m_{12}-\nu_1+(\lambda_j+\nu_1)e^{2\lambda_j(H_2-H_1)})$$

$$+\frac{1}{2}(\lambda_j m_{23}+\nu_2)(\Gamma^2+Q^2)e^{2\lambda_j(H_2-y_0)}(\lambda_j-\nu_1+(\lambda_j m_{12}+\nu_1)e^{2\lambda_j(H_2-H_1)})$$

$$-\frac{1}{2}(\lambda_j-\nu_2)(\Gamma^2+Q^2)e^{2\lambda_j y_0}(\lambda_j m_{12}-\nu_1+(\lambda_j+\nu_1)e^{2\lambda_j(H_2-H_1)})\Big]; \quad (1.11)$$

$$\Delta R_y^2 = -\frac{\rho_2}{2\pi}(\Gamma^2+Q^2)\int\limits_0^\infty [(\lambda m_{23}+\nu_2)(\lambda-\nu_1+(\lambda m_{12}+\nu_1)e^{2\lambda(H_2-H_1)})e^{2\lambda(H_2-y_0)}$$

$$+(\lambda-\nu_2)(\lambda m_{12}-\nu_1+(\lambda+\nu_1)e^{2\lambda(H_2-H_1)})e^{2\lambda y_0}]\frac{d\lambda}{T(\lambda)}. \quad (1.12)$$

Here ΔR_x^r and ΔR_y^r are additional forces to the generic Joukowski force acting on the vortex source.

The shape of the interface between D_k and D_{k+1} ($k = 1, 2$) is described by the formula

$$f_k(x) = -\frac{1}{\nu_k V_\infty}\operatorname{Re}\{m_{k\,k+1}^k \bar V_k(z) - m_{k\,k+1}^{k+1}\bar V_{k+1}(z)\}, \qquad z = x + iH_2(k-2),$$

where the complex velocities $\bar V_k(z)$ are found from (1.5)–(1.7) taking into account (1.8).

2. We introduce the dimensionless coefficients of wave drag $\Delta C_x = \Delta R_x H_2/(\rho_r\Gamma^2)$ and lift force $\Delta C_y = \Delta R_y H_2/(\rho_r\Gamma^2)$ of the vortex. For a particular case $\rho_3/\rho_2 = 0$ corresponding to vortex motion in a two-layer fluid bounded by a bottom from below and by a free surface from above, we consider the behavior of the coefficients ΔC_x and ΔC_y in a close neighborhood of the critical Froude numbers.

Performing operations described in [21] for Eqs. (1.9) and (1.10) in the case $r = 1$, we obtain the following values for the right and left limits:

$$\lim_{\mathrm{Fr}\to\mathrm{Fr}_{1,2*}-0}\Delta C_x = \frac{U_1}{U_2}, \qquad \lim_{\mathrm{Fr}\to\mathrm{Fr}_{1,2*}+0}\Delta C_x = 0,$$

$$U_1 = 3(h-h_1)^2 m_{12}^1(h_1(1-3m_{12})+4m_{12}\pm R),$$

$$U_2 = 2(1-h_1)^2(m_{12}^1 h_1((1-3m_{12})h_1\pm R)+2m_{12}(1-h_1)(1-5m_{12})), \quad (2.1)$$

$$R = 2\sqrt{m_{12}^1(m_{12}^1 h_1^2 - 4h_1 m_{12} + 4m_{12})};$$

$$\lim_{\mathrm{Fr}\to\mathrm{Fr}_{1,2*}+0}\Delta C_y = -\infty,$$

$$\lim_{\mathrm{Fr}\to\mathrm{Fr}_{1,2*}-0}\Delta C_y = -\frac{1}{2\pi}\Big[\frac{1}{2(h_1-h)}$$

$$+4(1-h)\frac{W_1}{W_2}+\int\limits_0^\infty\Big(\frac{g_1(\mu)}{g_2(\mu)}-2\frac{W_1}{W_2}\frac{(1+2\mu(h-1))e^{2\mu(1-h)}}{\mu^2}\Big)d\mu\Big]. \quad (2.2)$$

Here

$$g_1(\mu) = 2\sinh(2\mu(h_1 - h))(\mu + \bar{\nu}_{1,2*})(\mu + \bar{\nu}_{1,2*}m_{12} + m_{12}(\mu - \bar{\nu}_{1,2*})e^{2\mu})e^{2\mu(1-h_1)},$$

$$g_2(\mu) = (\mu - \bar{\nu}_{1,2*})(m_{12}(\mu + \bar{\nu}_{1,2*}) + (\mu - \bar{\nu}_{1,2*}m_{12})e^{2\mu})$$

$$+ (\mu + \bar{\nu}_{1,2*})(\mu + \bar{\nu}_{1,2*}m_{12} + m_{12}(\mu - \bar{\nu}_{1,2*})e^{2\mu})e^{2\mu(1-h_1)},$$

$$W_1 = -3(h - h_1)m_{12}^1(h_1(1 - 3m_{12}) + 4m_{12}^1 \pm R),$$

$$W_2 = (1 - h_1)^2(2m_{12}(1 - 5m_{12})(1 - h_1) \pm m_{12}^1 h_1 R + m_{12}^1(1 - 3m_{12})h_1^2),$$

$$h = \frac{y_0}{H_2}, \qquad h_1 = \frac{H_1}{H_2}, \qquad \bar{\nu} = \nu H_2, \qquad \bar{\nu}_{1,2*} = \frac{1}{\mathrm{Fr}_{1,2*}}.$$

Similarly, using the same operations for Eqs. (1.11) and (1.12) for $r = 2$, we obtain the limiting values ΔC_x and ΔC_y in a close neighborhood of the critical Froude numbers:

$$\lim_{\mathrm{Fr}\to\mathrm{Fr}_{1,2*}-0} \Delta C_x = \frac{U_1}{U_2}, \qquad \lim_{\mathrm{Fr}\to\mathrm{Fr}_{1,2*}+0} \Delta C_x = 0,$$

$$U_1 = -3(4h(h_1 - 2)m_{12}^1 m_{12} - 2h^2 m_{12}^1(m_{12}^1 h_1 - 2m_{12})$$

$$+ m_{12}(h_1(1 - 3m_{12}) + 4m_{12}) \pm (2hm_{12} - m_{12} - h^2 m_{12}^1)R), \tag{2.3}$$

$$U_2 = 2(m_{12}^1 h_1((1 - 3m_{12})h_1 \pm R) - 2m_{12}(h_1 - 1)(1 - 5m_{12}));$$

$$\lim_{\mathrm{Fr}\to\mathrm{Fr}_{1,2*}+0} \Delta C_y = \pm\infty,$$

$$\lim_{\mathrm{Fr}\to\mathrm{Fr}_{1,2*}-0} \Delta C_y = -\frac{1}{2\pi}\left[\frac{1}{2h} - 4h\frac{W_1}{W_2} + \int_0^\infty \left(\frac{g_1(\mu)}{g_2(\mu)} - 2\frac{W_1}{W_2}\frac{(1 + 2\mu h)e^{-2\mu h}}{\mu^2}\right)d\mu\right], \tag{2.4}$$

where

$$g_1(\mu) = 2(\mu\sinh(2\mu h) - \bar{\nu}_{1,2*}\cosh(2\mu h))(m_{12}(\mu - \bar{\nu}_{1,2*}) + (\mu + \bar{\nu}_{1,2*}m_{12})e^{2\mu(1-h_1)})$$

$$+ 2\bar{\nu}_{1,2*}(\mu - \bar{\nu}_{1,2*}m_{12} + m_{12}(\mu + \bar{\nu}_{1,2*})e^{2\mu(1-h_1)})e^{2\mu(1-h)},$$

$$g_2(\mu) = (\mu - \bar{\nu}_{1,2*})(m_{12}(\mu + \bar{\nu}_{1,2*}) + (\mu - \bar{\nu}_{1,2*}m_{12})e^{2\mu})$$

$$+ (\mu + \bar{\nu}_{1,2*})(\mu + \bar{\nu}_{1,2*}m_{12} + m_{12}(\mu - \bar{\nu}_{1,2*})e^{2\mu})e^{2\mu(1-h_1)},$$

$$W_1 = -3(m_{12}^1(hh_1 + (hh_1 - 2(2h + h_1) + 4)m_{12}) \pm (hm_{12}^1 - m_{12})R),$$

$$W_2 = h_1^2 m_{12}^1(1 - 3m_{12}) + 2m_{12}(1 - 5m_{12})(h_1 - 1) \pm h_1 m_{12}^1 R.$$

The upper sign (plus) in these expressions corresponds to the limiting transition to Fr_{1*} and the lower sign (minus) to Fr_{2*}.

An analysis of the limiting values of the coefficients ΔC_x and ΔC_y in the neighborhood of the critical Froude numbers for the problem of motion of a vortex source in a two-layer fluid bounded by a bottom from below and by a free surface from above allows us to find a discontinuity of the first kind in the wave drag and a discontinuity of the second kind in the lift force.

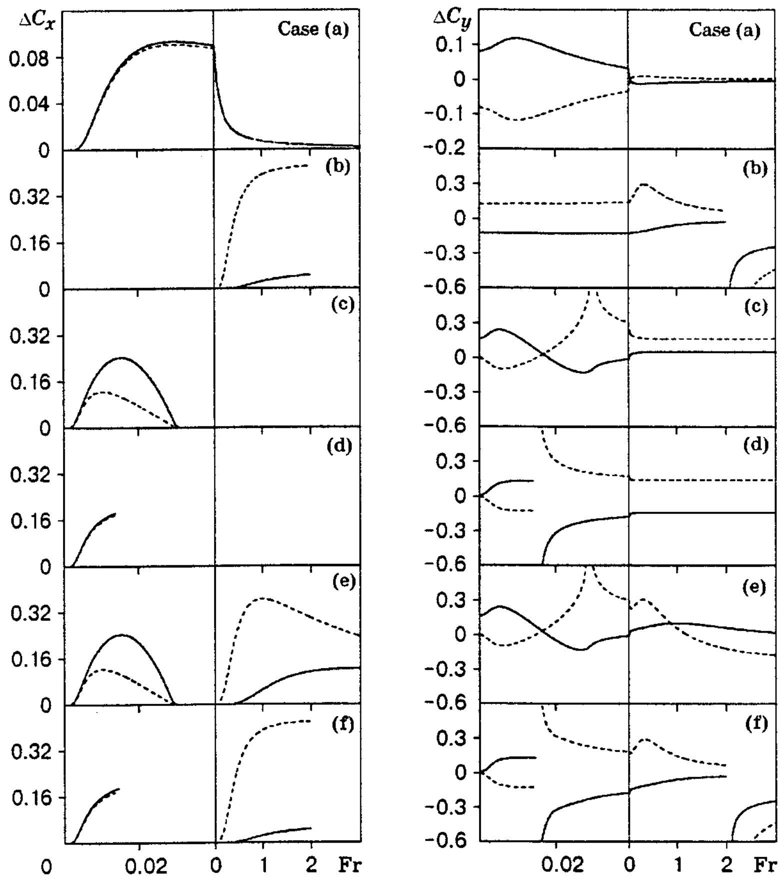

Fig. 1. Hydrodynamic loads acting on the vortex, which performs a uniform motion in a multilayer fluid: the solid and dashed curves are the results for vortices located in the layers D_1 and D_2, respectively.

3. Based on (1.9)–(1.12), an algorithm for calculation of the hydrodynamic loads ΔC_x and ΔC_y acting on the vortex was developed.

The following cases of vortex motion were considered:

Case (a): in a two-layer fluid ($\rho_2/\rho_1 = 0.970\,874$, $\rho_3/\rho_2 = 1$, and $H_1/H_2 = \infty$);

Case (b): under a free surface of a fluid of finite depth ($\rho_2/\rho_1 = 1$, $\rho_3/\rho_2 = 0$, and $H_1/H_2 = 2$);

Case (c): in a two-layer fluid under a solid lid ($\rho_2/\rho_1 = 0.970\,874$, $\rho_2/\rho_3 = 0$, and $H_1/H_2 = \infty$);

Case (d): in a two-layer fluid bounded by a solid straight channel ($\rho_2/\rho_1 = 0.970\,874$, $\rho_2/\rho_3 = 0$, and $H_1/H_2 = 2$);

Case (e): in a two-layer fluid under a free surface ($\rho_2/\rho_1 = 0.970\,874$, $\rho_3/\rho_2 = 0$, and $H_1/H_2 = \infty$);

Case (f): in a two-layer fluid bounded by a free surface from above and by a bottom from below ($\rho_2/\rho_1 = 0.970\,874$, $\rho_3/\rho_2 = 0$, and $H_1/H_2 = 2$).

The dimensionless parameter of the vortex depth h was 1.5 and 0.5 in the cases of vortex motion in the layers D_1 and D_2, respectively.

The calculation results for ΔC_x and ΔC_y for cases (a)–(f) are shown in Fig. 1. The critical Froude numbers for cases (b)–(e) are $\mathrm{Fr}_{1*} = 2.000000$, 0.030000, 0.014778, and 0.029126, respectively; the critical Froude numbers in case (f) are $\mathrm{Fr}_{1*} = 0.014\,671$ and $\mathrm{Fr}_{2*} = 1.985\,329$. Case (a) has no critical Froude numbers, which is attributed to the existence of internal waves for all values of the parameters of the problem. This problem was studied in more detail in [22].

Case (b) has only one critical Froude number. The hydrodynamic loads have a discontinuity at the transition through Fr_{1*}. The character of these discontinuities was established analytically in [21].

In cases (c) and (e), at the transition through Fr_{1*}, the coefficient ΔC_x remains continuous, and ΔC_y is continuous for $r = 1$ and has a discontinuity of the second kind for $r = 2$ (see Fig. 1).

In cases (d) and (f), at the transition through the critical Froude numbers, the coefficients ΔC_x and ΔC_y have discontinuities of the first and second kind, respectively (see Fig. 1). There is only one value of the critical Froude number in case (d) and two values in case (f).

In cases (b)–(e), internal waves are generated only for $\mathrm{Fr} < \mathrm{Fr}_{1*}$ (there is no wave drag for $\mathrm{Fr} > \mathrm{Fr}_{1*}$). In case (f), internal wave prevail for $\mathrm{Fr} < \mathrm{Fr}_{1*}$ and surface waves prevail for $\mathrm{Fr}_{1*} < \mathrm{Fr} < \mathrm{Fr}_{2*}$; for $\mathrm{Fr} > \mathrm{Fr}_{2*}$, no waves on the free surface and at the interface are formed, and we have $\Delta C_x = 0$.

The character of discontinuities in case (f) was established analytically based on system (2.1)–(2.4).

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